A SURVEY OF AUTOMATIC COMPUTERS
ANALOG AND DIGITAL

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INTRODUCTION

In some contexts, the term computer is interpreted to include any device that will perform the task of information processing. Most people think of computers in a more restricted way, as referring to aids for the manipulation of numbers in the solution of mathematical problems. A point of view in between these two seems appropriate here.

Talk and publicity about "thinking machines" and "giant brains" have obscured the fact that engineers have been using computers in industry for decades in the form of automatic process controllers. The theory, design principles and fields of usefulness of the various forms of automatic controllers commonly used in the process industry, are discussed. ~~fully covered in the preceding parts of Section 5.~~

Some of the art developed in connection with "scientific" computers (as they are sometimes called in order to distinguish them from controllers) is well established in the instrumentation engineer's bag of tricks if, indeed, it was not there first. It is the task of this section and the allied material in Section 8 to collect and classify some of the computer techniques ~~not as yet in wide use~~ in the field of instrumentation and control engineering, are described.

THE CLASSIFICATION OF COMPUTER TYPES

ANALOG-DIGITAL

The qualifiers "analog" and "digital" are frequently used to divide computers into two broad classes. In an analog machine, the magnitude of a variable is represented by the measured value of a single continuous quantity. In a digital machine, several separate measurements are made. Each measurement represents one digit of the number describing the magnitude of the variable.

Analog computers are based on the principle that the behavior of a given system can be imitated by another system in which measurements and changes are relatively easy. In other terms, the mathematical statement of the behavior of the system under study can be formulated in equations we are often unable to solve. These equations can be used to describe the behavior of an analogous system which is more convenient to work with, merely by changing the interpretation of the symbols in the mathematical expression. The behavior of the original system can be inferred from that of the test system.

In short, an analog computer measures, a digital computer counts and obeys logical rules exactly.

PRECISION

Suppose it is necessary to obtain the solution of a problem with an error no greater than o percent. (Throughout this section, the term "problem" should be interpreted to relate directly to a control function as well as to its mathematical formulation). In general, much greater precision will be required of the individual steps or partial solutions if the over-all result is to be

good to within one percent. If the operation of taking the difference between two large quantities is involved the importance of errors will be magnified.

The precision that may be obtained with analog equipment is determined by our ability to measure and control the physical quantities used. A large mechanical differential analyzer can achieve a precision better than one part in 100,000 for the elementary operations. Electrical analog computers are, at best, only one tenth as precise. Process controllers and recorders are good to about one part in 250.

Digital devices, on the other hand, may be made as precise as is desired by carrying enough digits. Each digit is represented by a magnitude of a physical quantity that need be measured and controlled only accurately enough to distinguish between its distinct values. In a decimal machine, equipment that may be in error by a few percent, is adequate to allow the ten states or values to be satisfactorily represented. The separate digits of a number can then be determined by duplication of equipment or by using the same equipment sequentially and distinguishing between the digits on a time sharing basis. The length to which digital precision may be carried is illustrated by the news that we now know the value of pi to an accuracy better than one part in 10^{2000}. By way of comparison, the total number of elementary particles in the universe is estimated at 10^{78}.

The relatively simple analog devices that are satisfactory in control functions today, will need to be augmented by more sophisticated techniques as the problems grow in complexity and

scope. In some cases the use of digital methods seems appropriate—see Section 8.

Special Purpose - General Purpose

Another way to classify various computer types, is to consider what they can do rather than how they do it. Each system can be graded according to its relative ability to handle a wide class of problems; this is a measure of its mathematical power. A well designed proportional controller with reset and derivitive features can be used in a great many applications by adjusting its parameters. However, in the sense of the terms as used here, it must be placed near the special purpose end of the scale. It cannot, for example, be used to do accounting.

The term "general purpose" is most distinctly typified by digital computers. The human being—desk calculator combination is a general purpose computer in that, in principle, any kind of problem that can be described in mathematical terms can be solved by digital means. This may be a completely impractical and grotesquely uneconomical method for finding the solution. It is absurd to think of mechanizing the jobs now done by automatic regulators in this manner; not only because of the cost, but because the solution time is an extremely important factor. The term "in principle" asserts the idea that a digital computer can solve the problem of what-to-do-to-the-fuel-valve-if-the-boiler-temperature-goes-up. Such a problem is a very simple one mathematically. A human operator—desk calculator computer could (if properly instructed as to the computing rules or program) solve the problem stated above, after it has been enriched with complications like prevention of smoke, changes in fuel quality, scaling in the

boiler, variations in steam load, etc. What-to-do, when finally calculated, would probably come too late to keep the safeties down. The modern "universal" electronic digital computer is a contrivance which can do the same sort of thing as the operator and desk calculator, but 10,000 times faster. In the literature of computing machines, the term "general purpose" is usually reserved, somewhat reverently, for such machines.

Speed

Allusion has been made to the last property to be used to classify various computer systems. It is perhaps the most important - - Speed. This is painfully evident if a computer is being used directly in a control loop in a "real time" application. A solution based on stale data is usually worse than useless - - it will produce hunting. Speed as it is used here, is a measure of the time to obtain a solution to the problem. The basic operating times of the large automatic electronic digital computers are measured in microseconds (10^{-6} seconds). However, in considering the solution of problems described by means of differential equations (as most dynamic control problems must be) this speed is not sufficient for some real time applications. The many thousands of steps required to obtain a solution, make even the fastest digital computers (such as the MIT Whirlwind) slower than the largest and most powerful analog computers (such as the RCA Typhoon) for problems of this type.

Figure 1 is an attempt to place some of the computer systems to be described later, and some of those that are already familiar, in a three dimensional "computer space".

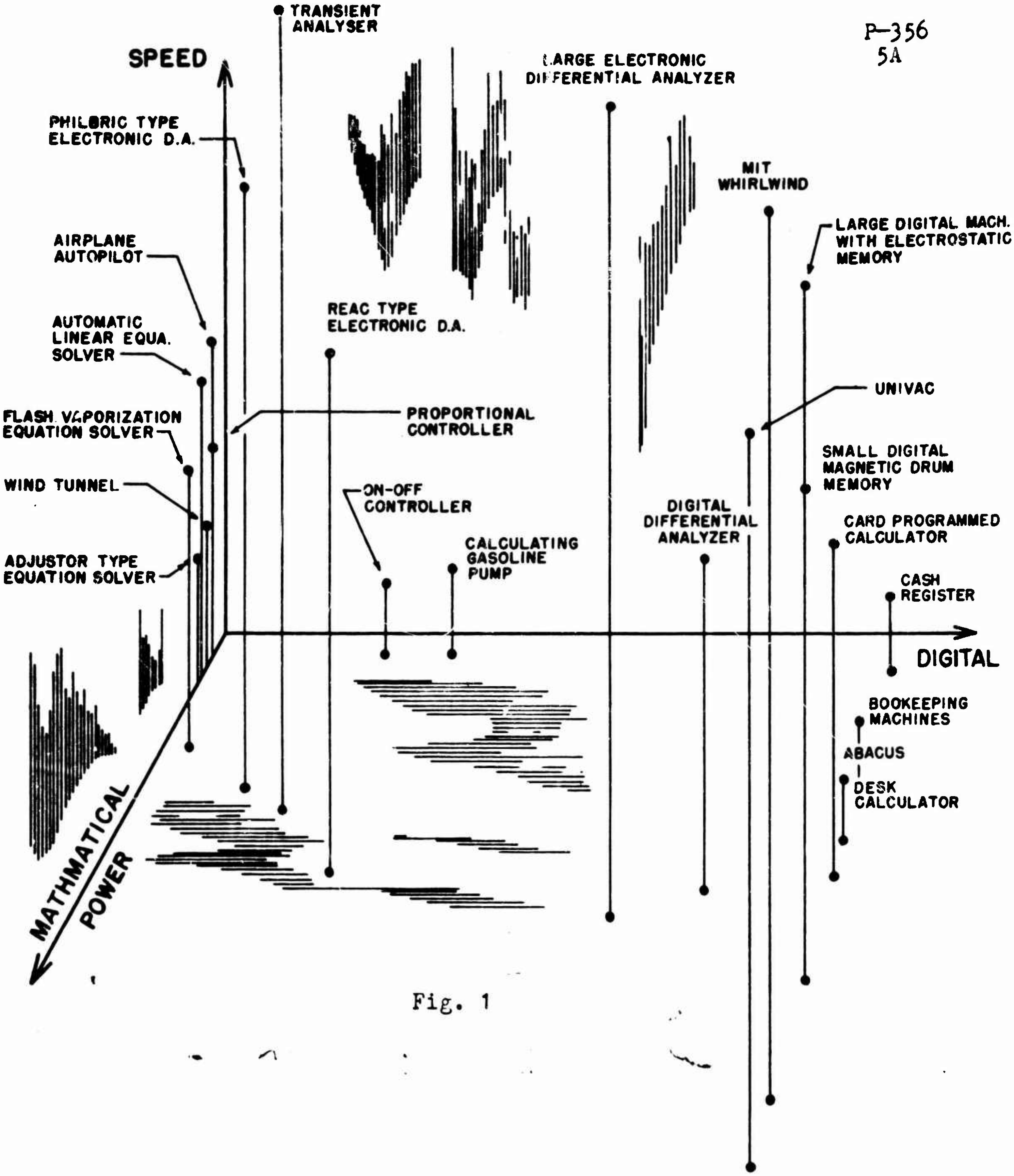


Fig. 1

Comparison of computer systems on the basis of

X axis mathematical power (special purpose → general purpose)

Y axis method of operation (analog → digital)

Z axis speed (slow to fast).

The origin represents a slow, analog, special purpose machine.

One other attribute that all computers have is, alas, cost. With some obvious exceptions like the wind tunnel and the abacus, cost is roughly correlated with distance from the origin of the plot in Figure 1.

Examples of Analog Computer Systems

Analog computers have assumed several different basic forms. They may be classified as (a) scale models, (b) "direct" analog (e.g. network analyzer) and (c) "mathematical" analog (e.g. differential analyzer).

Scale Models

Examples of the scale model class are the pilot plant and wind tunnel. The chief advantage of the scale model system is that the difficulty of mathematical formulation of the problem is almost completely sidestepped. If serious nonlinear and multiple-valued relations are present, mathematical description can be extremely difficult. There are pitfalls connected with establishing the correct scaling ratios. Simple extrapolation from model results to full scale performance can be disasterous. The principle disadvantage of the scale model method is its cost. A secondary problem is the relatively long time required to produce a solution. More detailed considerations of pilot plant operation and the use of the mathematical analog simulator as a substitute for part of the pilot plant are to be found in Section 8.

Direct Analog Computers

The soap bubble film analogy to stress flow and the rubber membrane-ball bearing model used to study electron trajectories

in vacuum tubes are direct mechanical analog computers close to the scale model concept. The more generally applicable electric analog computer is based on the observation that the same equations which describe many dynamic mechanical, acoustic, thermal or fluid systems can also be used to describe equivalent or analogous electrical systems. This sort of approach to the design of non-electrical systems is in part due to the development of an impressive body of mathematical analytical know-how for the use of the electrical engineer.

In the direct analog computer, the physical components of the system under investigation bear a direct correspondence to the electrical resistors, condensers, and inductors of the computer. The correspondence can be chosen in different ways. One common system is illustrated in Table 1.

Electrical	Mechanical	Thermal	Fluid
Resistance	Resistance (dash pot)	Resistance	Resistance (restriction)
Capacitance	Compliance (spring)	Capacitance	(tank)
Inductance	Mass	—	Inertia
Voltage	Force	Temperature	Pressure
Current	Linear Velocity	Heat flow	Flow
Charge	Displacement	Heat stored	Quantity

The DC network analyzer, introduced over 30 years ago, is the most elementary device in this class. It consists of a large array (typically about 150) of adjustable electrical resistances, one or more regulated sources of DC potential, the necessary meters,

and a switchboard to allow the components to be interconnected. These devices are still paying their way in the solution of short circuit current problems in electrical power distribution systems and in the solution of <u>steady state</u> heat flow problems. In the latter, the thermal system is divided into small compartments or "lumps". The thermal resistance (to heat flow) of each lump is replaced by an electrical resistance (to current flow). The electrical potential applied at a given point represents the temperature of the source. The temperature at any other point can then be determined by using a voltmeter.

Most interesting problems are dynamic rather than static. The AC network analyzer brings time into the picture by including capacitors and inductors. For thermal problems, the inductors are superfluous and thermal analyzers do not include them. To continue the example started above, each lump would now include both resistors and a capacitor to simulate the corresponding dynamic thermal properties.

The more generalized AC network analyzer was first introduced to handle problems in the distribution of electrical power. For this reason the large installations manufactured by General Electric and Westinghouse have many specialized components representing transmission line sections, generators, etc.

The electrical industry has put AC network analyzers to many uses not anticipated at the time of their original construction. Some examples are the solution of the Schrodinger wave equation for various simple atomic and molecular structures, the solution of fluid mechanics problems involving trans-sonic

compressible flow and various electro-mechanical problems involving vibration and stability.

In the AC network analyzer, the voltages and currents at various points in the array are measured in the steady state. The driving energy is approximately constant in amplitude and frequency, although it may be changed from run to run.

It was not until the introduction of the idea of the transient analyzer that wide interest was generated outside the electrical industry. In the transient analyzer, the electrical network of resistors, inductors and capacitors is excited by the sudden application of currents and/or voltages under the control of a bank of continuously running sychronous switches. The result of disturbing the network is then measured with a cathode ray oscilloscope. Oil pool analyzers and thermal analyzers are specialized computers of the transient analyzer class.

For many years, analyzers were used to solve problems that were linear and with constant coefficients. Such problems can be handled by classical mathematical methods of analysis. Analyzers provided a means for avoiding the tedium of lengthy calculations. The transient analyzer allows one to introduce such mathematically unruly, but practically very important things as violent nonlinearities, discontinuities and multiple valued functional relationships. The values of the network can be suddenly changed in midstream by means of the synchronous switches, and backlash or hysteresis effects can be introduced. The CRT, mask and photo cell technique (described in connection with the electronic differential

analyzer) is used to introduce nonlinear functional relationships. For example, actual empirical test data for the flow vs control pressure of a control valve or the force vs displacement of a nonlinear spring or vibration absorber can be used directly in the machine without the necessity of fitting a mathematical expression to the data. It is possible to produce reasonably good models of thermo-dynamic situations involving freezing, evaporation and combustion. Here it is necessary to use electronically-controlled circuit elements in which the effective electrical parameters are dependent on the time history of the voltage and current applied to that element.

Problems involving heat, fluid, or mechanical stress flow in more than one dimension can be rigorously described only by means of partial differential equations. Very little is known about solving these equations when nonlinear terms are present. Approximate solutions are obtained by replacing the partial differential equations with difference equations which involve nothing more mathematically onerous than total derivitives. The trouble is that there are so many of them. The principle of dividing a solid chunk of material into little pieces and considering each of these as a homogeneous single part is the physical equivalent of the mathematical simplification.

The network analyzer approach is well suited to problems which require a great many equations to describe the system performance. It is the best present method for attacking a problem with over 100 degrees of freedom where many different parameter

values must be tried. They are very often used as design synthesizers. That is, a network model of a proposed system is set up on the machine. The performance of the model is observed as the values of the components in the design are slowly changed.

Time enters into the choice of the particular conversion ratios between (a) the electrical quantities on the computer and (b) the physical quantities in the system under study. This means that the time scale on the computer may be vastly different from "real time". For example, one electronic reservoir analyzer presents the entire production history of an oil pool each 0.004 seconds. The effects of variations in pumping schedule may be seen immediately. On the other hand, a corresponding extension of the time scale would be used in the investigation of phenomena where the real time behavior is extremely rapid such as explosion investigations. A large transient network analyzer can obtain and display the solution of a problem representable by over 100 simultaneous differential equations (many of which may be non-linear) in times of the order of 0.1 second.

The values of the components in a transient analyzer are usually adjustable in about one-tenth percent steps. The solutions ordinarily have an uncertainty of one to ten percent. This is adequate accuracy, however, for a very large class of engineering problems.

The cost of a large modern transient analyzer lies in the range $100,000 to $300,000. Smaller and more specialized direct analog computers have been built in many locations. Oil pool

analyzers use an electrical resistance—capacitor analogy to the corresponding properties of the reservoir.

Radio quality resistors and capacitors may be measured and calibrated easily and they cost very little. A very respectable model of many transient heat flow problems can be wired for, say, $100 in parts and in a few days time. Experimentation on a model of this type can replace many weeks of arduous calculation.

Mathematical-Analog Computers

In direct analog machines just considered, the individual components of the computer can be related directly to the individual parts of the system under study. Where the system is essentially continuous, it is considered to be divided into parts. Each part has its counterpart in the electrical network.

The "mathematical"-analog computer uses a different principle. As before, the system must be described mathematically. When the computer is set up for a given problem, there is a correspondence between the components of the computer and the mathematical terms and operations of the equations describing the given system. Here the analogy is more abstract than in the "direct" case. In order to accomplish this correspondence, a mathematical analog computer must contain devices which will perform the operations of addition, subtraction, multiplication, division, the insertion of arbitrary functional relationships, and most important of all, integration. It is the ability to form the integral of one variable quantity with respect to another that gives these machines their power. This is also the source of the

name differential analyzer since an integrator "analyzes" a differential or derivitive term. It is not necessary to work out an equivalent electrical network, a task which often calls for considerable experience and ingenuity.

The differential analyzer is the most elegant of the mathematical analog computers. The (non-differential) equation solver may be considered to be a special case of the differential analyzer.

Computers of this class have appeared in two basic forms (a) those depending principally on mechanical motion or fluid flow and (b) those using electrical equipment.

The pneumatic controller is the most highly refined fluid computing mechanism in wide use. There are many applications of mechanical motion computing mechanisms. The index and pointer linkages used in many regulators and recorders, is a familiar example. The military establishment has favored mechanical systems as, for example in the famous Norden bombsight. The specialized practical principles of design of bar linkage computers are well documented.

The Mechanical Differential Analyzer

Some of the devices used in mechanical computing mechanisms will be illustrated by considering one form of the mechanical differential analyzer. In this machine, the variables are represented by the rotation of shafts. The direction of a shaft rotation corresponds to the algebraic sign of a quantity. Addition and subtraction are obtained by means of a differential gear, Fig. 2.

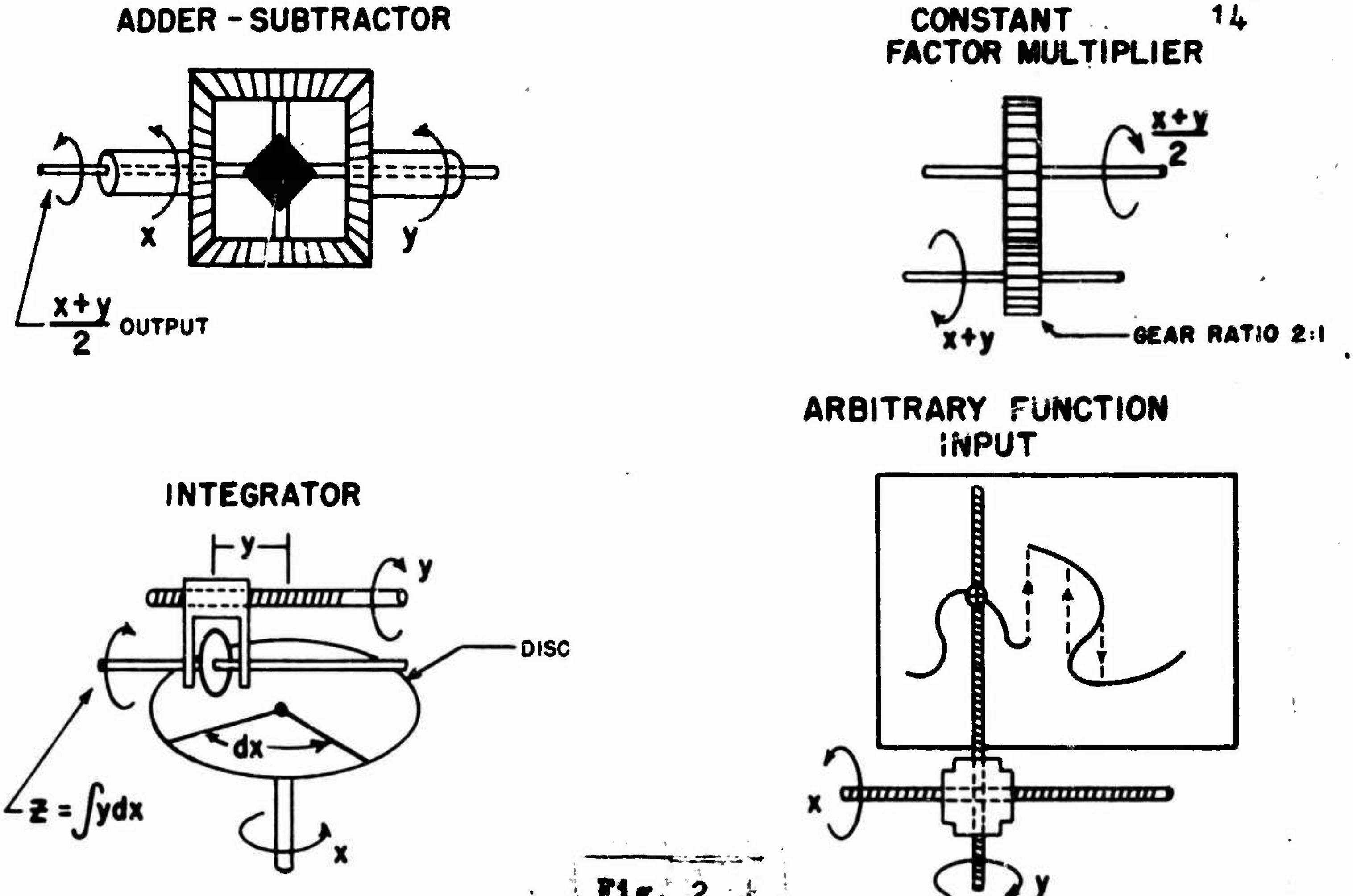


Fig. 2

The two inputs, called x and y, are applied to the sleeve shafts. If the x shaft rotates 14.3 turns and the y shaft rotates 7.5 turns in the directions indicated by the arrows, the output shaft will rotate $\frac{x+y}{2}$, or 10.9 turns. If the direction of the y shaft were reversed, the output would be $\frac{x-y}{2} = 3.4$ turns. A simple gear pair is used to obtain the quantity kz, where k is a constant. The output of one differential can become the input of another to build up the sum $[(x+y) + z]$.

The process of integration can be mechanized by any continuously variable ratio transmission mechanism. The wheel and disk device is the most accurate. To form $z = \int y\,dx$, y is applied to the lead screw which moves the integrating wheel to a radial position proportional to y. When y = 0 the integrating wheel is at the exact center of the disk. If the value of y is reasonably

constant for a small rotation of the disk (dx), then the output shaft will rotate through a small angle (dz) which is proportional to the product $y \cdot dx$. Thus $z = \int dz = \int y\, dx$. To prevent slipping, a servo mechanism must isolate the torque load of the rest of the machine from the output shaft of the integrator. In the General Electric machines, the integrating wheel contains a sheet of Polaroid material. The output shaft of the servo follow-up is coupled to a similar Polaroid disk which rotates parallel to the integrating wheel. An optical system is arranged to give an error signal to the servo whenever the output shaft angular position is not equal to that of the integrating wheel. A practical angular sensing device with less reflected drag is hard to imagine.

The process of setting up a mechanical differential analyzer may be illustrated by the following example. The product of two variable quantities is often mechanized by using two integrators and a differential gear adder to solve the equation

$$z = xy = \int y\, dx + \int x\, dy.$$

The input variables x and y are applied to the inputs of two integrators. The output shafts of the integrators are coupled to the two inputs of a differential gear adder.

Arbitrary functions are entered into the machine by a curve follower. Referring to Fig. 2, the shaft representing the dependent variable y (the output) is turned (either manually, or by means of a photo electric servo follow-up on some machines) to keep the reticle continuously over the plotted curve as the independent variable shaft x (the input) changes. The function

$y = f(x)$ may be discontinuous and/or multiple valued.

Specially prepared cams are used to insert arbitrary functions in special purpose mechanical analog computers. Functions of two independent variables have been handled by "three dimensional" cams.

A gear mechanism is essentially a digital device. Thus the precision obtainable with a mechanical differential analyzer is limited by the care used in the design of the integrators which are the principle source of internal error. This error can be as little as 0.001 percent of full scale for each integrator. The qualifier "full scale" implies something about the delicacy of the job of assigning the ratios between the number of turns of a shaft and the magnitude of the quantity represented by that shaft. This is referred to as the "scale factor". The accuracy with which the machine analog imitates the real situation behavior is usually quite sensitive to the form in which the describing equations are cast and to the care with which the scale factors are chosen. Often an order of magnitude reduction in error can be achieved by mathematical rearrangements of the form of the problem.

In any analog machine, the limit on the complexity of the problem that may be handled is determined by the amount of equipment available. The largest mechanical differential analyzers have about 20 integrators. This means that a system requiring more than, say, a dozen simultaneous nonlinear differential equations for an adequate description of its behavior cannot be investigated by any existing mechanical differential analyzer. The much less

accurate transient analyzer would not be taxed at all by a problem of this complexity assuming that a proper equivalent electrical circuit can be found.

The large mechanical differential analyzers which give answers with errors less than 0.1 percent cost over $200,000. Small machines for special problems using ball and disk integrators and no servomechanisms can cost less than $10,000 and produce results with errors of about one percent.

<u>Equation Solvers</u>

Between the mechanical differential analyzer and its electronic kin, there is a class of machines used to attack problems expressible in equations which do not involve derivatives or integrals. The Beckman Phase Equilibrium Computer, solves a non-linear algebraic equation. The Consolidated Engineering and the RCA equation solvers are electrical machines designed to handle linear equations. They all have the potentiometer multiplier as a common component.

If a voltage (AC or DC) is applied to a potentiometer (Fig. 3), then a fraction of that voltage is picked off by the sliding contact.

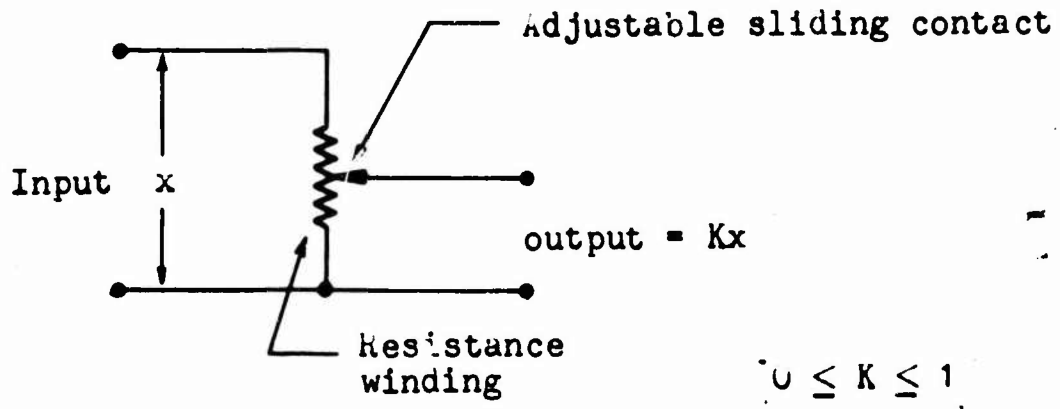


Fig. 3

One way to solve the equations

$$a_{11} x_1 + a_{12} x_2 + b_1 = 0,$$

$$a_{21} x_1 + a_{22} x_2 + b_2 = 0,$$

is to form voltages proportional to the terms and add them. Since it is always possible to make all of the a's and b's less than unity, the multiplications required in four of the terms may be made by potentiometers.

Some of the a's and b's might be negative. A potentiometer cannot produce a negative fraction of the input voltage. This difficulty can be met by providing equal and opposite voltages proportional to +x and −x. In Figure 4 the negative number difficulty is handled by providing each adjustable element with a reversing switch.

Voltages proportional to x_1 and x_2 are obtained from adjustable transformers, and applied to the a_{ij} coefficient potentiometers. The voltages representing the three terms of each equation are added together. A problem is put on the machine by setting the switches and potentiometers to represent the known values of the a's and b's. The solution (if one exists) is found by adjusting the voltages x_1 and x_2 such that the outputs of the adders are simultaneously zero.

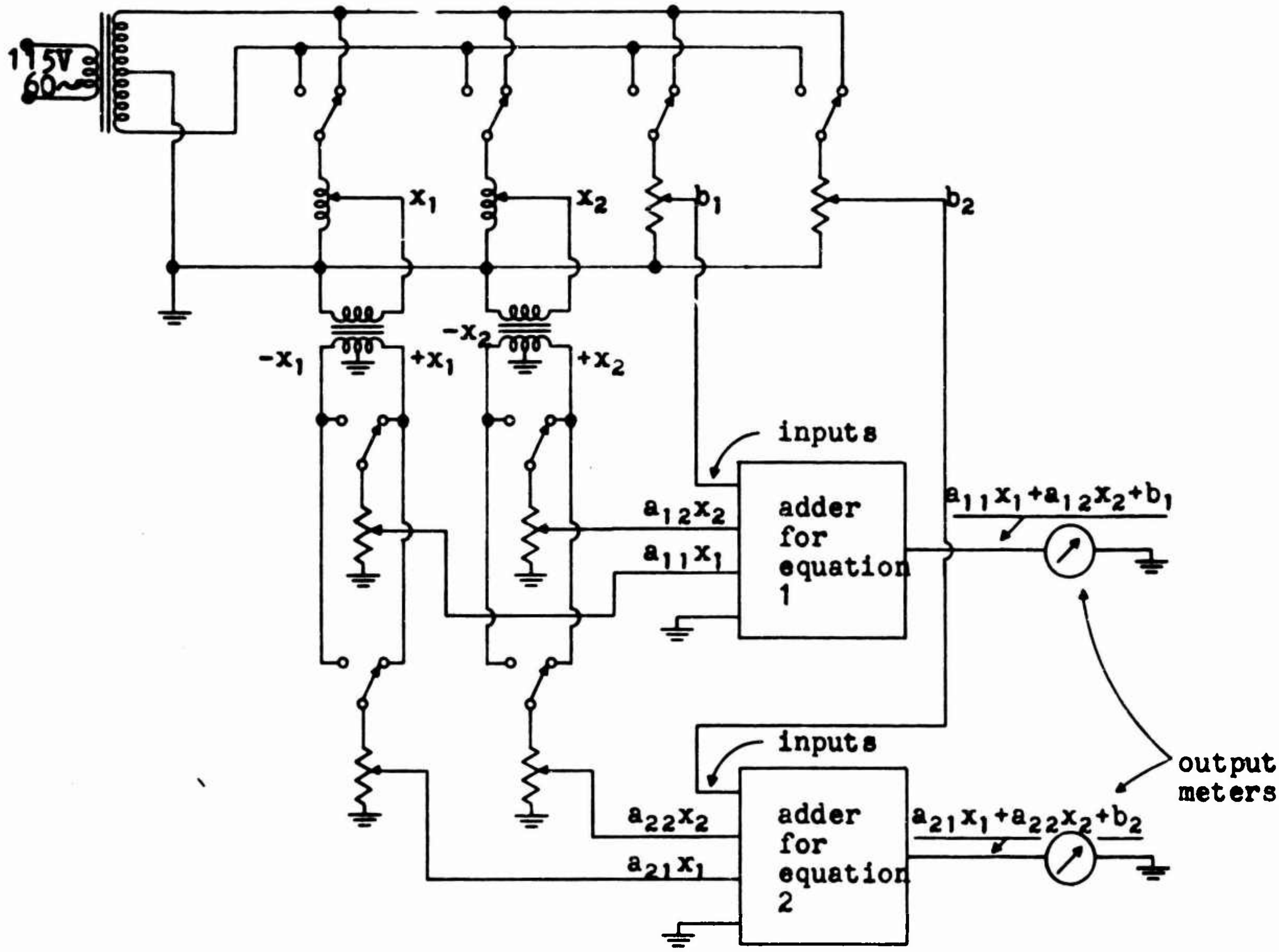


Fig. 4

To be useful, machines must have enough elements for say 10 equations in 10 unknowns. In the Consolidated Engineering and RCA machines, 110 potentiometers must be adjusted to insert the a's and b's for a problem in 10 equations and 10 unknowns. This may require about 30 minutes if the values are to be carefully checked against an accurate reference standard. Often, for example in spectrometer data analysis, solutions are required for many problems where the a's are the same from run to run and only the

b's need be changed. An analog equation solver has been designed by IBM whereby the a's and b's are set up by inserting a separate punched card in a special card reader for each equation plus one additional card and reader for the constant terms.

For most problems arising in industry, the process of successively adjusting the values of the voltages representing the x's will not take more than 10 minutes if the equations are properly arranged. Equation solvers that work by this principle are called "adjustor" types. The Consolidated Engineering, and IBM machines are of this class.

An "automatic" equation solver will display the solution (if it exists) immediately after the a's and b's are set. To explain one method used to accomplish this feat we are led to a discussion of the basic element of the electronic differential analyzer – the operational amplifier. The RCA equation solver is of the "automatic" variety and, although it uses AC voltages and operational amplifiers, it is exactly the same in principle as the DC amplifier equipment to be described.

Electronic Differential Analyzers

The basic idea of continuity of flow is applicable to electrical networks and may be stated as the sum of all the currents flowing into (and out of) a given junction of a network must be zero.

In close analogy to the flow of fluids in pipes, the flow of current through an electrical resistance is directly proportional to the voltage or potential difference (pressure drop) across the resistor and inversely proportional to the resistance. This is Ohm's law.

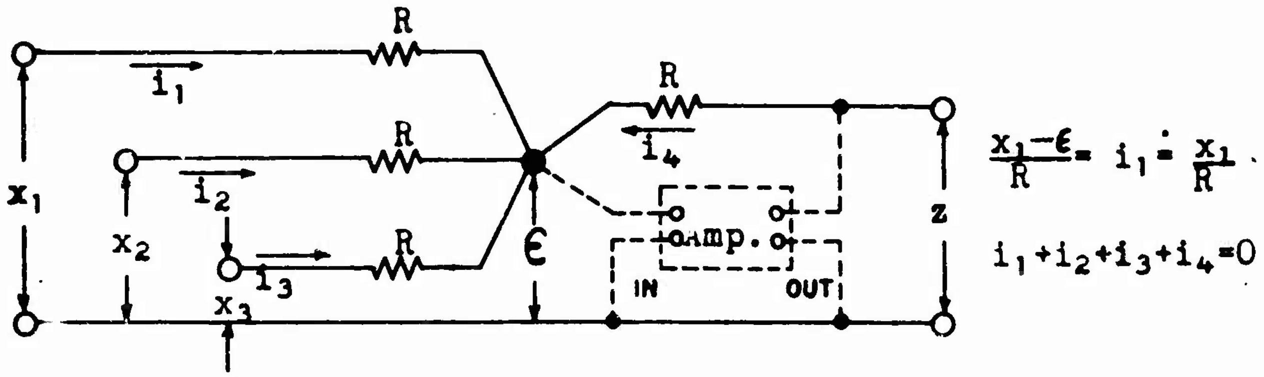


Fig. 5

Ignore the dotted amplifier for a moment and suppose, in Fig. 5, that, for some reason, the voltage ϵ turned out to be so close to zero that it could be neglected in determining the voltage across each of the resistors. The current sum expression would then be

$$\frac{x_1}{R} + \frac{x_2}{R} + \frac{x_3}{R} + \frac{z}{R} = 0, \tag{1}$$

or

$$x_1 + x_2 + x_3 = -z. \tag{2}$$

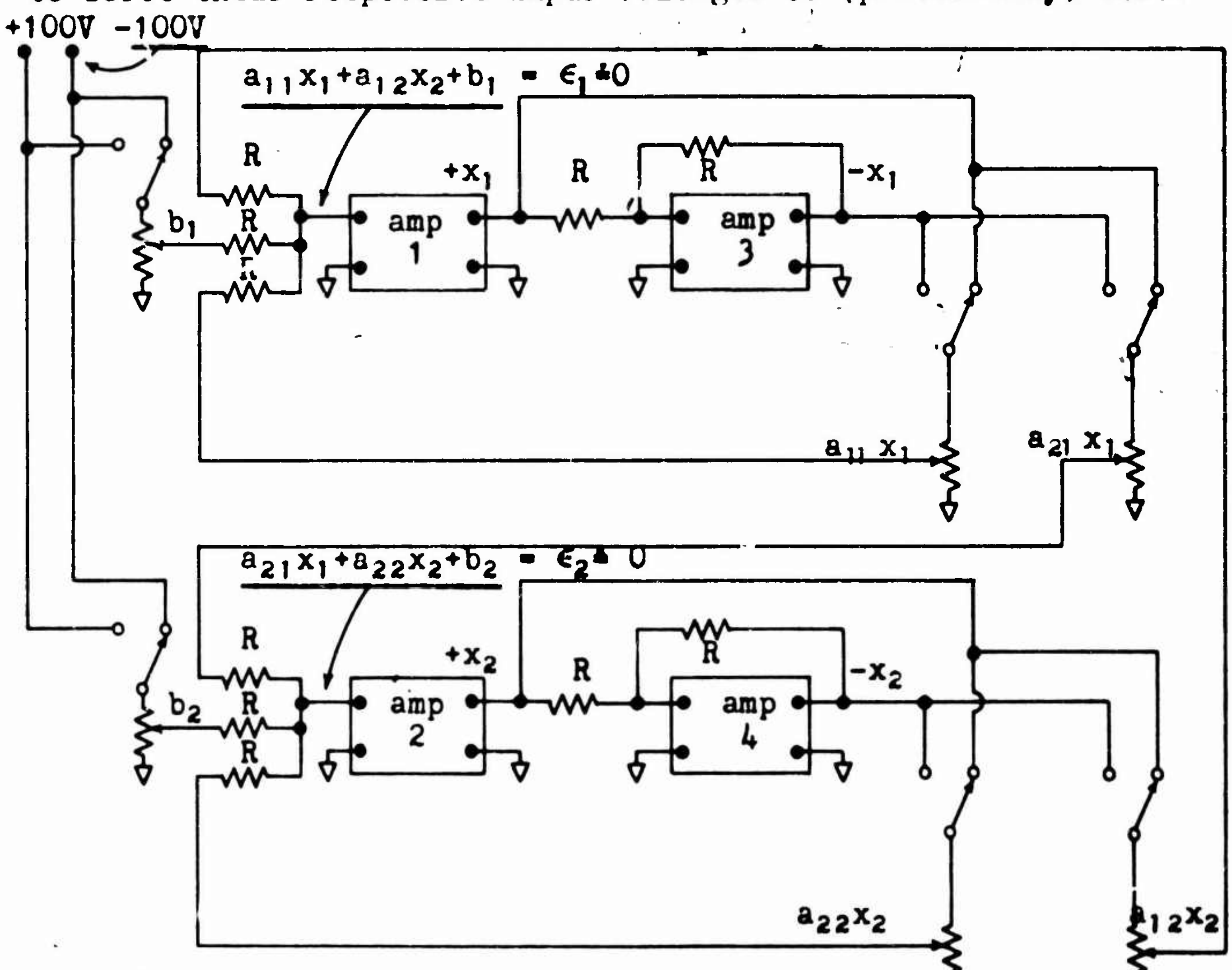
This state of affairs can be brought about by making ϵ the input to a very high gain amplifier and calling the output of the amplifier z. For definiteness, take an amplifier with a gain of -10^3, i.e. + 0.01 volt input will produce a -10 volt output. Assume that x_1, x_2 and x_3 are exactly +10, +20 and +30 volts respectively. The output will fall short of the correct value of -60 volts by just enough to leave a positive "error voltage" (ϵ) of about +0.06 volts, (the output divided by the gain). The larger the gain of the amplifier, the smaller will be the error. Gain values of 10^7 are common in modern DC amplifier machines.

One way to look at this arrangement is to think of the amplifier as a source of voltage of just the right magnitude and

sign to force a current back into the input junction that will (almost exactly) cancel the sum of the other currents.

This is what is known as a "summing" amplifier. (It is simultaneously an "inverting" amplifier since the algebraic sign is changed). The resistor connected between the output of an amplifier and its input is the feed-back resistor and the other resistors are input resistors.

Fig. 6 shows how summing amplifiers may be used as an automatic equation solver. Amplifiers 1 and 2 "solve" the two equations by producing just the right output voltages (x_1 and x_2) to force their respective input voltages to (practically) zero.



$R = 10^6$ ohms

Fig.6

Amplifier 3 and 4 with their input and feedback resistors operate as sign changes or inverters to produce an output of $-x$ for an input of $+x$.

An electrical condenser is a device which will store electrons much like water is stored in a tank. The potential difference across the terminals of a condenser is directly proportional to the time integral of the current or the total number of electrons (total charge) that has been allowed to flow into it and inversely proportional to the size of the condenser.

$$E = \frac{1}{C} \int i\,dt = \frac{Q}{C} \qquad (3)$$

where E is the potential difference in volts,

C is the capacity in farads,

Q is the charge in columbs,

i is the current in amperes and

t is the time in seconds.

If the feedback resistor of the summing amplifier is replaced by a condenser, the current balance condition is

$$\frac{x_1}{R} + \frac{x_2}{R} + \frac{x_3}{R} = -\,i_0 \qquad (4)$$

where i_0 is the current forced into the condenser by the amplifier.

Integrating both sides of (4) and using equation (3) gives

$$\frac{1}{RC} \int (x_1 + x_2 + x_3)\,dt = -\frac{1}{C} \int i_0\,dt = e_0 = -\,z \qquad (5)$$

which states that the magnitude of the output voltage is proportional to the integral (with respect to time) of the sum of the input voltages.

Electronic differential analyzers made by Beckman, Boeing,

Electronic Associates, Goodyear, Phillbric, RCA, Reeves and others, contain many basic high-gain DC amplifiers which may be connected to perform the operations of summing or integrating. Prices range from about $50 to $500 per amplifier.

Errors arise from (1) drift in resistor and condenser ratios and (2) drift in the balance and insufficient gain in the amplifiers. High quality wirewound resistors and plastic condensers have been developed which will allow individual component operations with errors less than 0.01 percent to be obtained. The bugaboo of drift that plagued wide-band high-gain DC amplifiers since their inception, has been completely dispelled by borrowing the idea of the chopper from instrumentation engineers. The wide-band input signal is divided. The low frequency components are passed through a contact modulated amplifier which has negligible drift. The high frequency components are passed through a conventional amplifier and the two outputs combined. Stable open loop gains of 10^7 with equivalent input drift of a few microvolts are common.

Two widely used methods for inserting arbitrary functions automatically are the non-linear potentiometer and the cathode ray tube, mask and photocell. If a given function is to be used many times, a specially prepared potentiometer may be purchased in which the voltage picked off at the arm is, say, sine x times the total voltage across the resistance winding where the slider has been moved proportional to x. Other non-linear potentiometer methods are available which allow the function to be changed easily. In the cathode ray tube, mask and photocell electric method, Fig. 7,

a mask representing the desired function is placed on the face of the cathode ray tube.

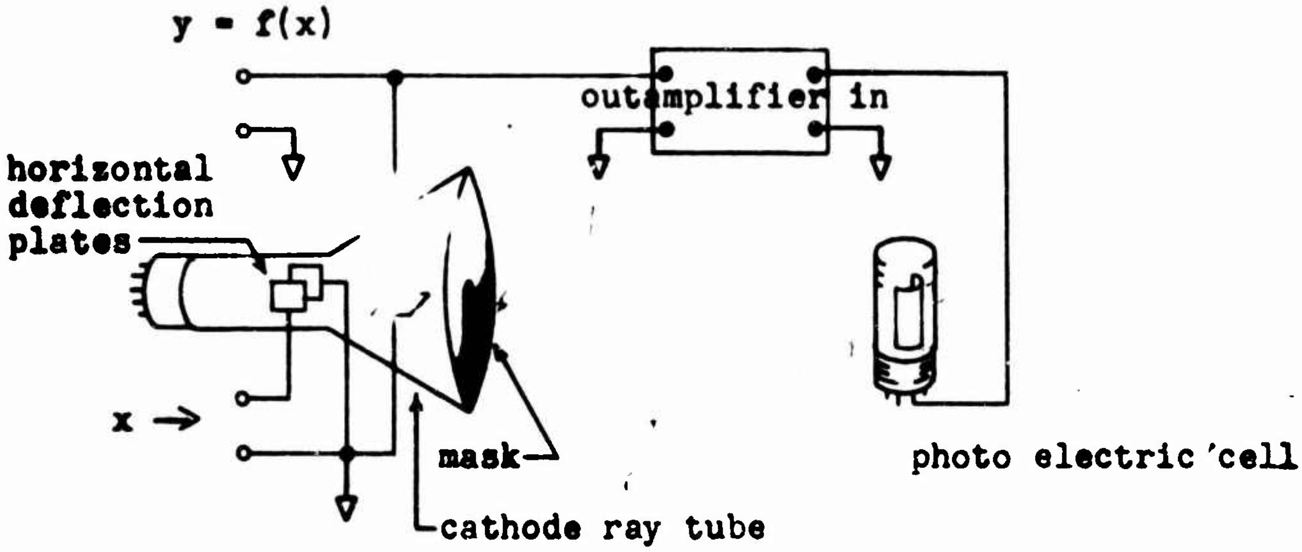


Fig.7

As the independent variable x (the input) positions the spot horizontally, the photo electric cell amplifier controls the vertical position to keep it just on the edge of the mask. The output voltage of the amplifier is proportional to the function, y = f(x), represented by the mask.

Table II is rough comparison of the two methods.

	Speed of Response	Percent of error	Cost
CRT	.01-.0001 sec. (electron beam)	1 to 1/2 percent	$1,000
N.L. Pot	5 - .1 sec. (involves mechanical motion)	1 to 0.1 percent	$100-1,000

Multiplication of one variable voltage by another is commonly accomplished with a "servo-multiplier". This is very much like a self balancing potentiometer used for thermocouple recording with extra potentiometers coupled to the output shaft.

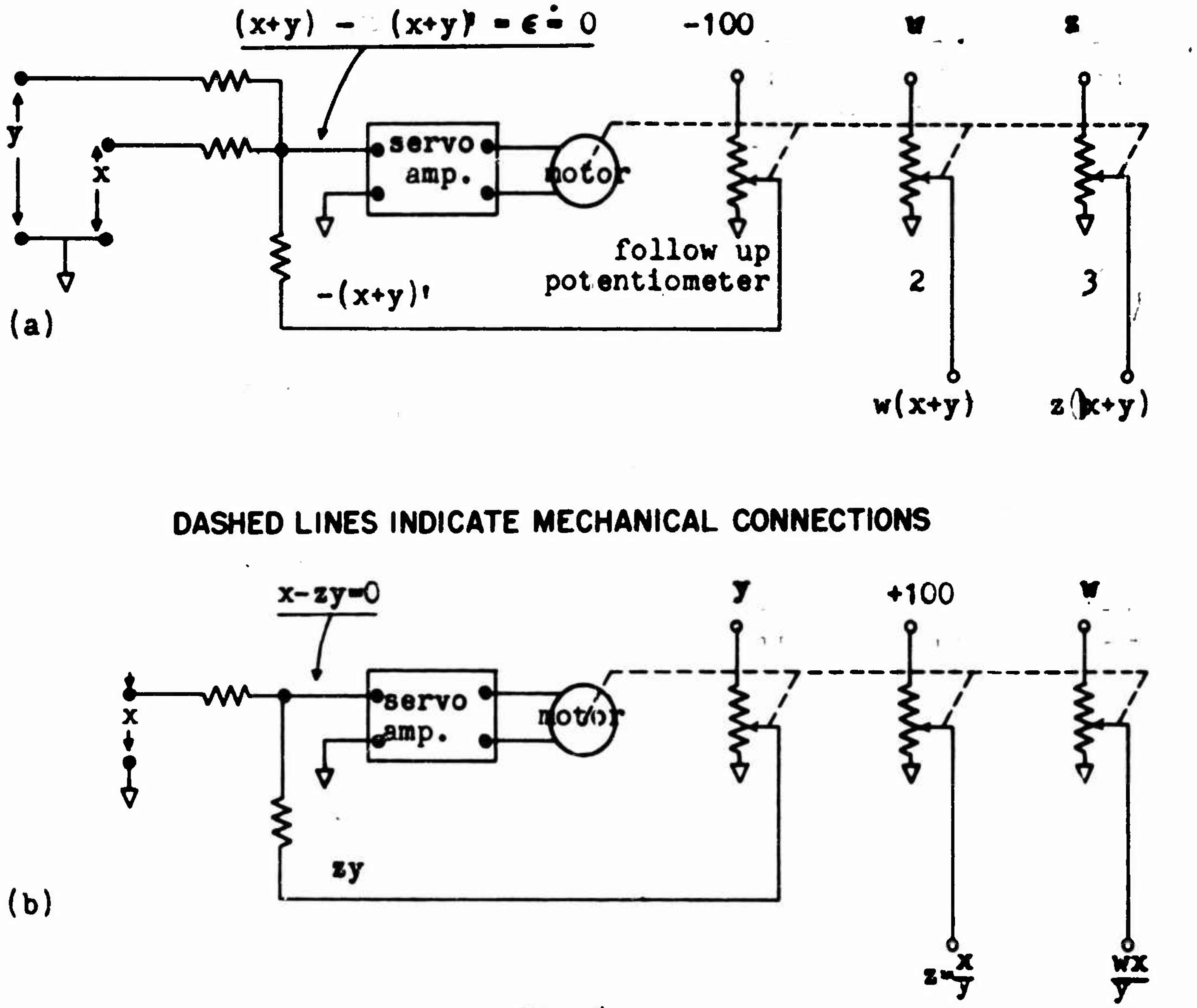


Fig.8

In Fig.8 (a) the servo amplifier solves the equation $(x + y) - (x+y)' = \epsilon \doteq 0$ by turning the shaft just enough to pick off $-(x+y)'$ volts from the follow-up potentiometer. Potentiometers 2 and 3 pick off a fraction $x+y$ of their total voltage to form the products $w(x+y)$ and $z(x+y)$. Division may be accomplished as shown in Fig.8 (b) where the servo amplifier solves the equation $x - zy = \epsilon \doteq 0$ by turning the shaft enough to pick off a fraction z of the total voltage (y) across the follow-up potentiometer.

Helical potentiometers are commonly used which allow products and quotients to be formed with an error of about 0.1 percent of full scale. The response time for this accuracy is typically

about 3 seconds.

Electronic multipliers with no moving parts are now available which combine analog and digital techniques. A series of pulses is produced in which the time duration is made proportional to one variable and the amplitude (and sign) is made proportional to the other variable. The area of the pulse is proportional to the product. Stabilized versions have errors less than 0.1 percent, do not drift or need adjustment, and respond in about 0.005 seconds.

It is possible to use AC instead of DC and perform the fundamental operations of a differential analyzer. The basic equipment manufactured by Arma and others, makes use of various forms of rotatable transformers. The errors are, in general, greater than those found in corresponding DC equipment. The reliability can be higher since in many situations vacuum tube amplifiers are not needed. This method is attractive for special purpose applioations and has been favored for many military uses.

The Digital Differential Analyzer

One of the advantages of some analog machines is their ability to form the integral of a variable. None of the "universal" or general purpose digital computers contain integration in their list of basic operations. Integration must be approximated by a series of many separate instructions calling for multiplications and additions. The digital differential analyzer is a specialized machine that works purely digitally internally and does perform the approximation to integration as a basic operation. The first of this class of machines was called Maddida.

Problems are prepared for it in the same direct manner used for an analog differential analyzer. The digital nature of the machine allows the user to choose between (a) relatively slow operation (one hour per solution) and obtain very precise results (error less than one part in 10^6 or (b) trade precision for speed.

The product of speed times accuracy is perhaps one-tenth that obtainable with the best DC electronic analogue machines.

Table III compares the 3 basic types of differential analyzers. The entries in the table are very approximate and show trends only.

TABLE III

Type	Number of integrators	Cost in $1,000.	Speed, seconds per solution	Percent error of integration
Mechanical (large scale)	20	200	1000	0.001
(ball and disk integrator, no servos)	10	10	600	0.1
Electronic (analogue)	300	1000	10	0.02
	20	15	10	0.1
	20	5	10	0.5
	10	5	0.1	0.1
(digital)	50	100	10^4 / 10	0.0001 / 0.1

AUTOMATIC DIGITAL COMPUTERS

Introduction

The desk calculator is a digital machine which requires human intervention for each basic mathematical operation. Usually, the computing program is prepared in advance, along with a work sheet containing the starting data and space for intermediate results. Standard punched card machines are automatic in that (1) data are read mechanically or electrically, (2) the calculation rules are set up in a control board and (3) the results are either printed in tabular form or punched into a different space in the cards as they pass through the machine. Each time the type of calculation is changed from, say, multiplications to additions, the wiring of the control board must be modified.

The desk calculator-operator combination is more flexible since the type of mathematical operation can be different for each step. However, the punched card machine (particularly if it uses electronic computing internally) is much faster, is less likely to make a mistake and rarely complains. In the IBM Card Programed Calculator and the famous IBM-Harvard Mark I computer, operations can change from step to step since instructions stating what is to be done, are fed to the machine along with the new numbers.

The operator-desk calculator combination still has the upper hand over the CPC and Mark I in one respect, but is matched by universal machines. In many situations, the instructions for the operator will depend on what happens as the calculation proceeds. For example, if, at a certain stage, the numbers are too small, follow procedure A, otherwise follow procedure B. The ability to have the choice between alternates in a sequence of instructions

depend on the results of the computation is the most important feature of a "universal" digital machine. Many choose to define this action as a rudimentary form of thinking. It is important to note that the machine proceeds sensibly only if it has been given instructions and, in the case of decisions or "branching operations", the course of action for each alternative must be completely spelled out. This characteristic is not unique to digital machines. Modern electrical analog machines include arrangements to allow the problem setup to be altered automatically depending on certain results of the computation. The CPC and Mark I computers cannot alter their instructions.

Since many electronic machines can add two 10 digit numbers in 10^{-4} seconds, it is absurd to think of using a keyboard to type in individual instructions and numbers. To take advantage of the high arithmetic speed, it is necessary to provide some means for making (a) the list of instructions that describe the problem and (b) the data to be operated upon available to the machine at comparable speeds.

In summary, an automatic digital computer may be considered to be universal if it has (1) internal storage for the instructions and starting data that characterize the problem and (2) the ability to modify the instructions according to the results as they develop.

Internal Organization

The block diagram of an automatic digital computer is most easily understood by considering the correspondence with an operator-desk calculator system. In Fig.9, the

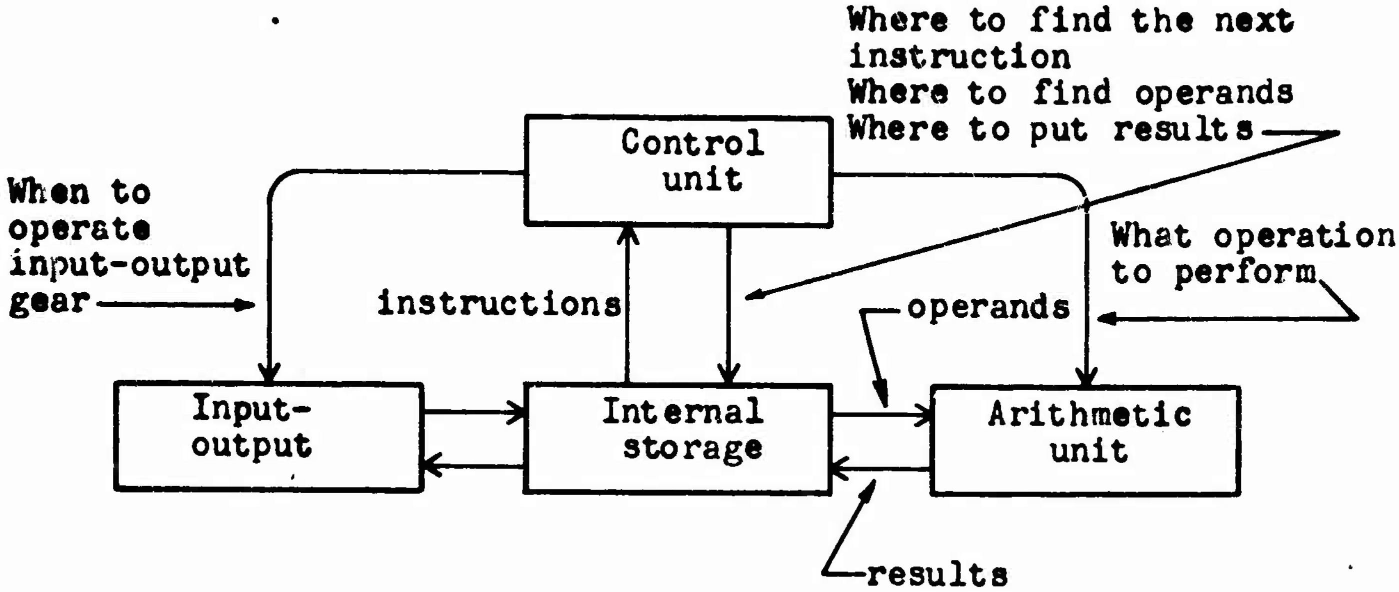


Fig.9

arithmetic unit is an electronic equivalent of the mechanical desk calculator. Since there are no moving parts except electrons, the basic operating times are measured in microseconds (10^{-6} seconds).

Internal Storage is the machine equivalent of the work pad, instruction list and mathematical tables and charts used by the operator. The control unit fulfills the role of the operator by (1) interpreting the instructions in proper order, (2) selecting the correct operands or numbers to be fed to the arithmetic unit from the storage unit, (3) issuing commands to the arithmetic unit to cause it to perform the operation called for in the instruction, (4) directing the recording of results in the storage and (5) calling for additional instructions or data or making the results available via the input-output devices. The duties of the control may always be rigorously described in complete detail by mathematical-logical expressions.

The control unit will respond sensibly to only a fixed number (usually between 10 and 100) of different input signals or instructions and will produce output signals (or commands) for the other sections of the machine. The control usually contains one counter to keep track of where it is in the sequence of instructions and another to keep track of the individual steps of the more complicated arithmetic operations like multiplication.

Universal digital machines are classified as serial or parallel, coded decimal or binary, single or multiple address. The first distinction describes the method used to transfer information between parts of the machine. In a serial machine, the digits (or characters) are sent over a single channel serially in time sequence. In a parallel machine, all digits are transferred simultaneously over separate parallel channels. Parallel machines contain more equipment but are faster than serial machines. To prepare problems for a machine, the user need never know whether it is serial or parallel internally, but the decimal or binary nature may be important to him. To understand this distinction, we must explain binary numbers.

The decimal number 9435 is shorthand for the sum $(9 \times 10^3)+(4 \times 10^2)+(3 \times 10^1)+(5 \times 10^0)$. The ten digits 0 through 9 are eligible as multipliers of powers of the "base" 10 in forming a decimal number. Some argue that our 10 fingers represent an unfortunate anatomical accident; 8 or perhaps 12 would have produced a simpler number system.

Binary (or base 2) numbers make use of only two digits, 0 and 1. In binary notation, 10110 is shorthand for (decimal equivalent) $(1 \times 2^4)+(0 \times 2^3)+(1 \times 2^2)+(1 \times 2^1)+(0 \times 2^0) = 16 + 0 + 4 + 2 + 0 = 22.$

In addition of binary numbers, 1 + 1 = 10 (0 and carry one).

Thus,

Binary		Decimal
10110	=	22
11011	=	27
110001	=	49

Binary notation was adopted in electronic computers because it is only necessary to have two possible values to represent a digit. A circuit is open or closed; a pulse of current is either present or absent at a given time.

Because we are accustomed to the decimal system, input numbers must be converted from decimal to binary notation before a binary machine can work on them; similarly, the binary results must be reconverted to decimal. In the coded decimal machine the conversion process is avoided and the simplicity of on-off circuit elements is retained. The ENIAC uses 10 binary units, (only one of which may be "on" at a given time) to represent each decimal digit. Several machines use only 4 elements with a different combination of off and on units to represent each decimal digit. Coded decimal machines are (1) more complex internally since decimal arithmetic is more complicated than binary and (2) less efficient in the sense that 4 binary units <u>can</u> take on 16 states, 6 of which are disregarded in a coded decimal arrangement.

There are modern programing methods which will cause a binary machine to do conversion and reconversion automatically and make the process quite painless for the user.

Binary operation is preferred when a digital machine is used in a real time-control operation. Analog <——> binary digital converters are used as input-output coupling means.

The distinction between single and multiple address describes the method used by the machine to interpret instructions. The inner mechanism of a machine may be simplified if the instructions and numbers are written in the same "language". A standard number of digits, which may represent either an instruction or an operand, is referred to as a "word".

A word may be called from or sent to a particular compartment in internal storage only if the "address" of the compartment is specified as part of the instruction. In a four address instruction, two of the addresses are used to specify the locations in the storage from which the two operands are to be drawn. The third address determines the location for the result. The control uses the fourth address to locate the next instruction word which is to be obeyed. The first three addresses control the flow of information between the storage and arithmetic units, while the fourth address regulates the storage to control path. Many machines use a three address instruction word and a counter to locate instructions which are placed in consecutive storage addresses.

In many calculations, the result of one operation is used as one of the operands for the next step. For example in computing 2xyz, xy might be formed first. Waste motion is eliminated if this preliminary result is left in the arithmetic unit and considered as one of the operands of the second step. The next instruction needs only one address and might be "multiply the number in the arithmetic unit by the number stored in address such and such".

(The computing program has arranged that address such and such contains the number z). Considerations of this kind led to the single address instruction machine. The number of digits that must be carried in a standard number word to assure the required computational precision, is usually great enough to allow the specification of two complete single address instructions in one word length. The space required to store a given computing program is about the same for any of these instruction addressing methods.

Like binary vs decimal, the arguments about the merits of single vs various multiple address schemes are not very exciting any more. Clever programs have been worked out which cause a single address machine to behave as if it were a multiple address machine. The "pseudo" multiple address orders are taken in and automatically translated into a series of basic single address instructions which the machine proceeds to obey in the standard manner. Such a program is called an "interpretive" sub-routine. By this means the effective "vocabulary" of instructions to which the machine can sensibly respond may be increased indefinitely.

Arithmetic and Control Elements

Electronic digital computers need mechanisms for only two basic functions - storage and switching. The particular forms used vary with the taste of the designer and also with constraints (such as cost, speed of operation required, etc.) placed upon him. One of the most widely used devices for static storage of one binary digit or "bit" is the vacuum tube "flip-flop" or "toggle". This device is a two-tube (or stage) direct-coupled amplifier with positive (regenerative) feedback. The feedback makes the amplifier

so nonlinear that only one of the tubes will pass current at a time. In the most basic form (Fig.10), if Tube 1 is conducting, the resulting plate current will produce a voltage drop in its plate resistor.

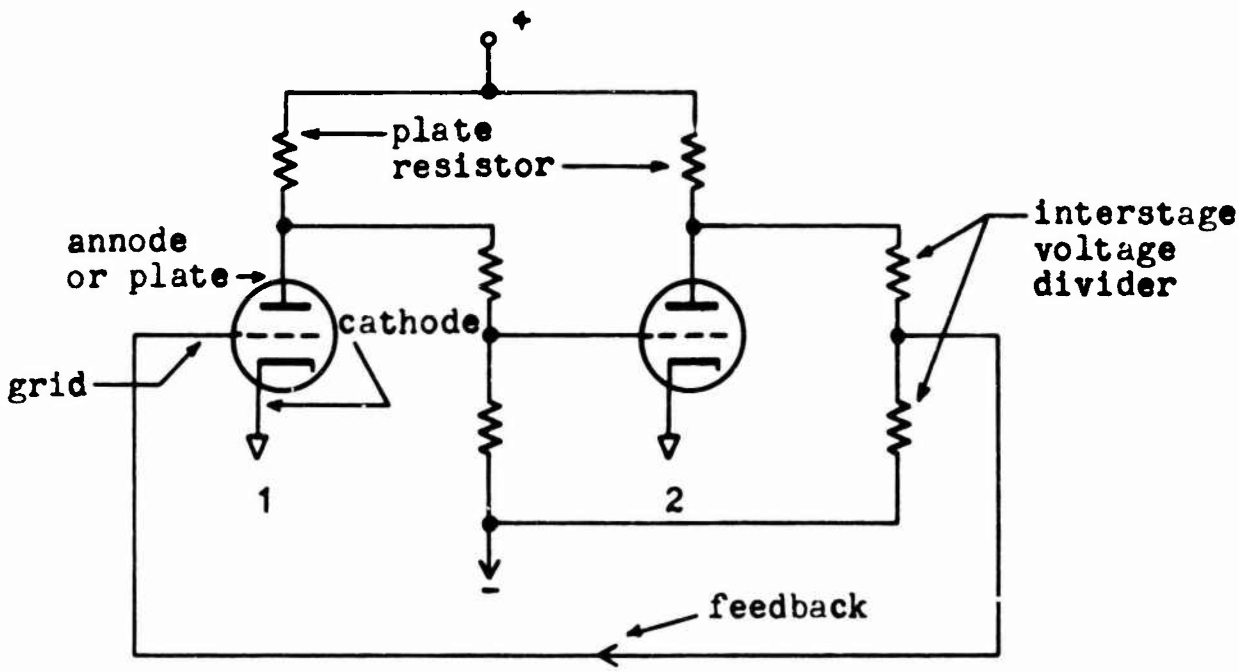


Fig.10

The first interstage voltage divider couples the drop in plate voltage to the grid of stage 2. This will prevent tube 2 from conducting since a negative voltage on the grid inhibits the flow of (negative electron) current. Tube 2 is said to be "biased to (or beyond) cut-off". The lack of plate current in tube 2 allows the second interstage voltage divider to present a positive voltage to the grid of Table 1. This assures the flow of plate current assumed at the start. The circuit is completely symmetric; a similar argument will show that if tube 2 is on, tube 1 must be off. The state of two or more toggles can be compared and caused to produce an output by means of switching networks or "gates" which commonly consist of arrays of electronic diodes.

In Fig.11, if three inputs connected

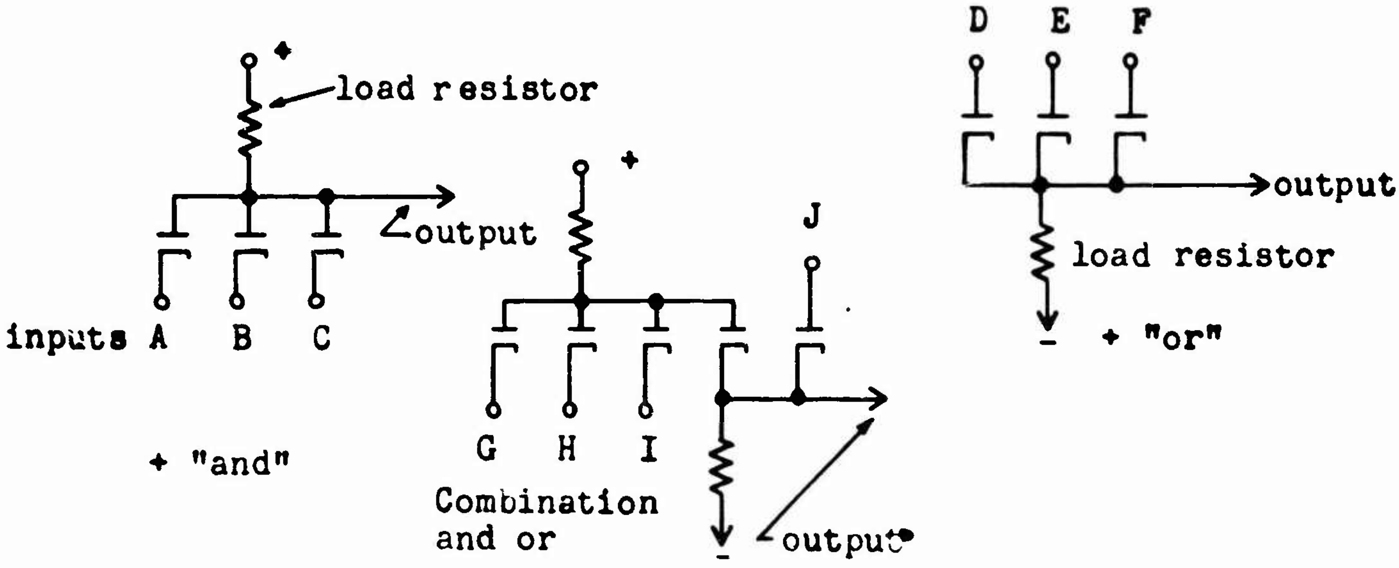


Fig.11

to A, B and C are all positive then the output will be positive. If one or more of the inputs is negative, the resulting diode current will pull the output negative. This is called a "positive and" gate since inputs A and B and C must be positive in order to obtain a positive output. In the "positive or" example, if D or E or F is positive, the output will be positive. In the mixed gate structure the output will be positive if J is positive or, if G and H and I are positive. Germanium crystal diodes developed for use in radar circuits are commonly used instead of vacuum tube diodes to construct elaborate switching networks. The SEAC computer contains over 15,000 diodes.

Internal High Speed Storage Devices

The internal high speed storage unit has assumed many forms. A fair portion of the enormous population of over 18,000 vacuum tubes used in the ENIAC is devoted to internal storage. Later machines have used more subtle and, at first glance, some rather astonishing methods. There are two basic classes. The magnetic drum and mercury tank devices typify one system in which the infor-

mation continuously circulates past an observation station. If an instruction specifies an address that has just passed, the machine must wait, sometimes for thousands of microseconds, until the word becomes available again. The faster machines use an electrostatic "random access" system in which any address can be reached in a definite time which is typically from 10 to 30 microseconds.

A magnetic drum storage (Fig.12) consists basically of a continuously rotating cylinder coated with magnetizable material and an array of magnetic recording "heads".

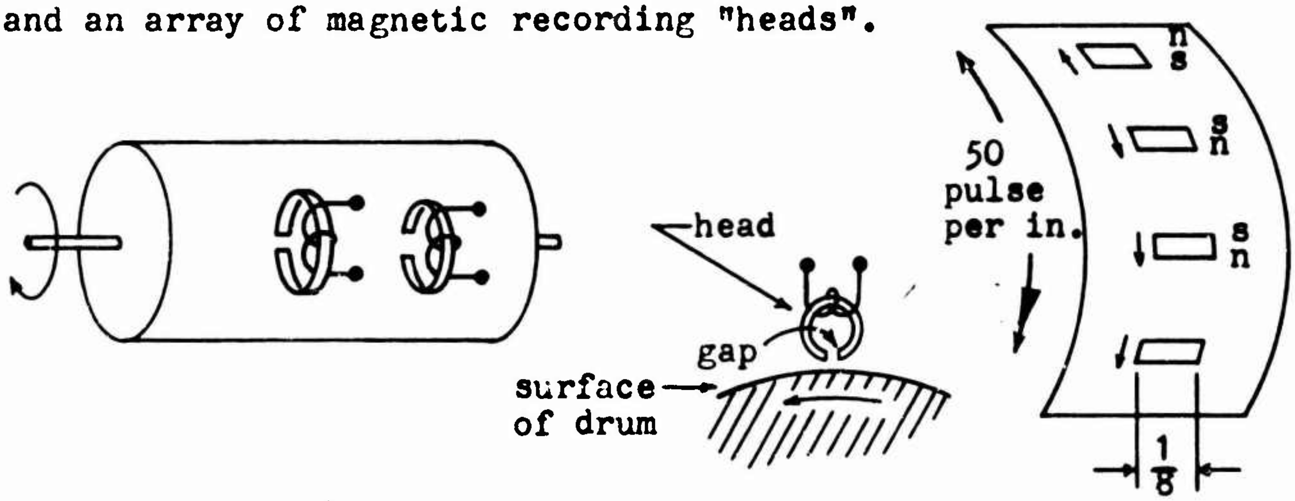


Fig.12

In one method used to record information, accurately timed short pulses of current are sent through the coil. The direction of the current determines the polarity (North-South or South-North) of each small permanent magnet produced on the surface of the drum under the gap in the head. The two different orientations correspond to the binary numbers 0 and 1. Each head controls a "track" on the drum. The density of information is typically 20 to 100 separate permanent magnetic dipoles per inch along a track which is perhaps one-eighth inch wide. The information remains permanently recorded until another series of current pulses writes

new information, erasing the old pattern in the process.

The same head is usually used to read information out of storage. As the previously magnetized areas pass rapidly under the gap of the head, a pulse of voltage is produced across the terminals of the coil. The polarity (+ or -) of each pulse depends on the direction (N-S or S-N) of the correspondingly magnetic dipole.

For a given drum diameter, the RPM is chosen to provide a surface speed that is ordinarily 1,000 to 2,000 inches per second. The total mechanical runout is less than 0.001 and is often about 0.0002 inches. The heads are spaced 0.001 to 0.002 inches from the surface. The circulation time for information stored on a drum is determined by its RPM, which, in turn, depends on the diameter and the capacity of storage. A typical value being about 20,000 microseconds. Various schemes using more than one head per track have been developed to reduce the waiting time.

The mercury tank is a faster circulating storage system of the "delay line" type in which the information is stored in a traveling acoustic wave pattern in a column of liquid mercury.

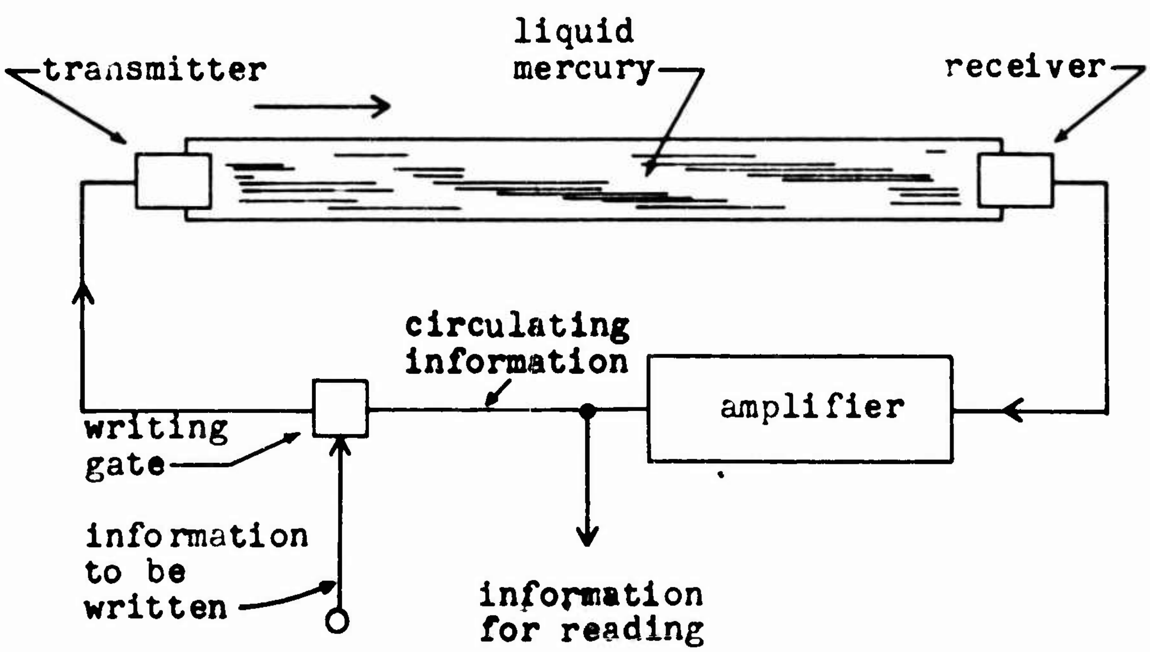


Fig.13

Information is entered into the storage by supplying a series of timed pulses to the acoustic transmitter. The resulting sound waves travel through the mercury at 0.145 centimeters per microsecond and are picked up by the receiver. An amplifier reshapes and builds up the strength of the received pulses and sends them back to the transmitter. Once a pattern of pulses is started it will continue to circulate until the path is broken to allow new information to be stored. The storage may be read by sampling the output of the amplifier as the pulses stream by. In the SEAC the mercury is contained in glass tubes about one inch in diameter and 20 inches long. Many such "tanks" are commonly used for a given storage system. The velocity of acoustic propagation changes with temperature and therefore a mercury storage system is carefully thermostated in order to maintain synchronism of delay between the various units. From 100 to 1,000 binary digits may be stored in each tank. The circulation time is typically 1/2 to 1 microsecond per bit stored. Several machines have used similar circulating storage systems in which electromagnetic waves traveling through an electrical delay line are substituted for the acoustic waves traveling through the mercury acoustic delay line.

The separate tanks in a delay line storage system correspond to the tracks on a magnetic drum. An address must specify the particular circulating loop and the position in the loop to pick out a single word. Both systems are "synchronous" and depend on the time sequence of pulses. All operations are controlled with reference to a master timing unit containing a "clock" and pulse distributor. Clock pulses are commonly derived from a separate track in the magnetic drum system.

In an electrostatic storage system, information is recorded by charging or discharging a capacitor, and read later by examining the state of charge. A separate capacitor is used for each binary digit stored. The problem of arranging a large array of capacitors and providing a fast and efficient way to switch onto each one individually has been solved by using a cathode ray tube similar to (but much smaller than) a television picture tube. The screen is divided into between 250 and 1,000 individual areas each one of which is considered to be a storage capacitor. The electron beam may be directed to any elementary area and, by proper time and position modulation, one of two charge distributions established. A fine mesh screen fastened to the outside face of the tube serves as a reading signal pickup electrode. Information is read out of a particular cell by directing the electron beam to the chosen area.

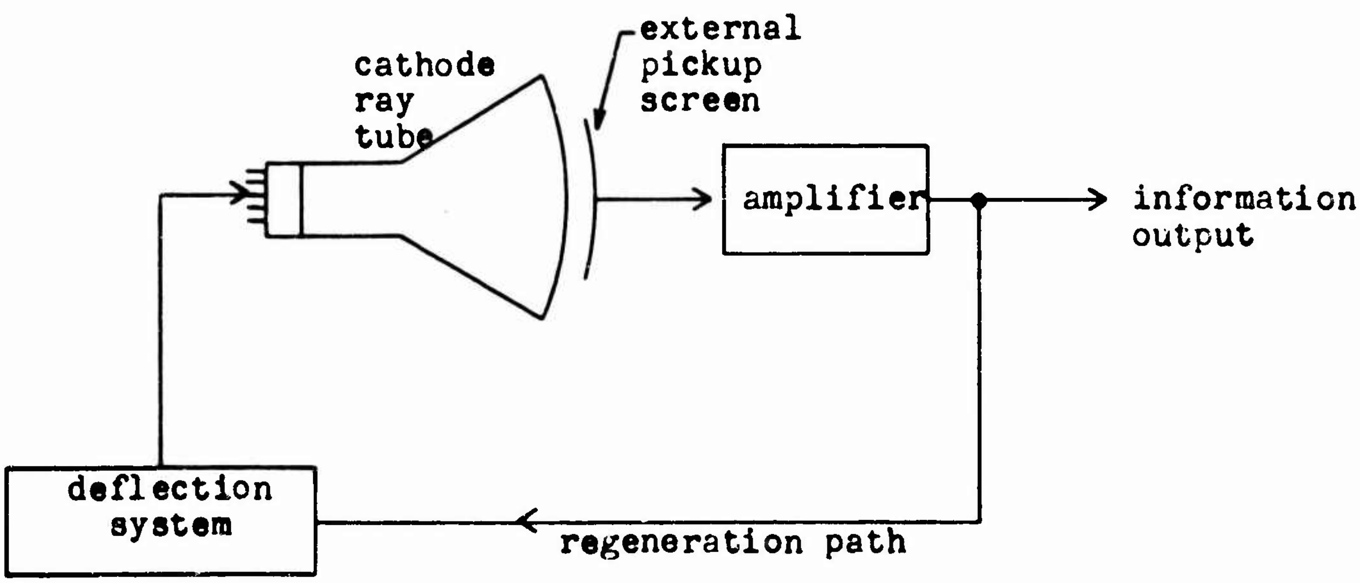


Fig.14
Cathode Ray Tube Electrostatic Storage

The charge on the individual capacitors gradually leaks off and the information would be lost if it were not continuously regenerated, one spot at a time. Professor Williams, first to use the system, has said that the storage remembers by continually muttering to itself.

The speed of this form of memory is derived from the use of electrical selection of an element and the lack of any mechanical (or acoustic) motion.

Static (i.e. no mechanical motion) magnetic storage systems of large capacity and low cost using non-metallic ferromagnetic materials are under development. Access times less than 5 microseconds and capacities of 10,000 words seem to be practical goals. Table IV is a rough comparison of three types of internal storage.

Table IV

Type	Magnetic Drum	Mercury Tank	Electrostatic
Average time (in microseconds) to locate and read or write in a specified cell. (Typical)	2000 to 25000	200	25
Binary digits per channel. (Max.)	4000	1000	1000
Total approximate binary digits stored in a complete system. (Max.)	10^6	10^5	5×10^4

Input Output Devices

Analog <——> digital converters described in the section on Data Handling, must be used as input-output means to incorporate digital computers into control loops. A cathode ray tube driven by a high speed digital-to-analog converter has been used on some computers to give a qualitative graphical output display.

Usually, purely digital input and output means are used. The approximate speeds of the various typical mechanisms are summarized in Table V.

Table V

	Speed	
	Decimal digits/sec.	Binary digits/sec.
5-hole punched tape	20 200 *	100 1000 *
Punched card (1)	200 2000 *	1400 14000 *
Magnetic wire	2500	10000
6-channel magnetic tape	10000	60000
Automatic typewriter	10	10
Line-at-a-time printer (2)	100	100
Matrix of dots printer (2)	1200	1200

* Optical type

(1) 70 columns used for information

(2) 40 characters per line

Several of the printers can handle alphabetic characters as well as numerals. A matrix of dots printer produces characters by printing combinations of small dots in positions selected from a rectangular array which is typically five positions wide and seven positions high. Fig.15 shows the appearance of two different characters when round dots are used. Some machines print square dots.

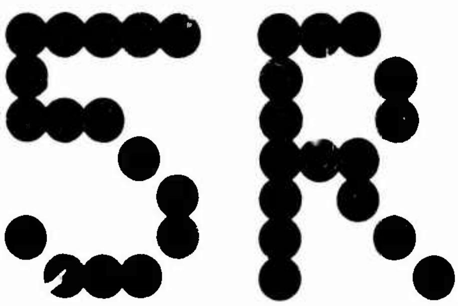


Fig.15

Eastman Kodak has demonstrated a matrix of dots mechanical printer that will produce about 400 characters per second with each printhead. A 60 printhead device of this type could turn out as many printed characters as the Encyclopedia Britannica contains in about two and one-half hours.

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